

MAT 115: Finite Math for Computer Science

Problem Set 2

Due: 02/22/2019

Instructions:

I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: /140

Problem 1 Proof: 10pts

For all integers a, b, m, n if $m \% d = a$ and $n \% d = b$, does that mean $(m + n) \% d = (a + b)$? If yes please prove; if no, please give counter example (assuming that $d \in \mathbb{Z}^+$)

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Problem 2 Quantifiers: 20pts

Determine the truth value of each of these statements, given $x \in \mathbb{R}$. Furthermore, if the truth value is false (that is the statement is not true), please show a counter example.

(a) $\exists x(x^2 = 2)$

(b) $\exists x(x^2 = -1)$

(c) $\forall x(x^2 + 1 \geq 2)$

(d) $\forall x(x^2 \neq x)$

Problem 3 Proof: 10pts

Please show that for any integer m and n , $m^2 - n^2$ is even **if only if** $m - n$ is even.

(a) Given any integer m and n , if $m^2 - n^2$ is even, then $m - n$ is even

(b) Given any integer m and n , if $m - n$ is even, then $m^2 - n^2$ is even

Problem 4 Proof: 10pts

Given a function $f(x) = (x - 3)(x - 5)(x - 7)$ is odd for all interger x if only if x is even. Here if only if means $p \rightarrow q$ and $q \rightarrow p$. Please show

(a) $f(x)$ is odd \rightarrow x is even

(b) x is even $\rightarrow f(x)$ is odd

Problem 5 Predicate Logic: 10pts

$D = \{1, 3, 4, 5, 121, 144, 196, 256, 289, 441\}$, $S(x) = (\sqrt{x} \in \mathbb{Z} \wedge \sqrt{x} + 2 \in \mathbb{P})$ where \mathbb{P} is the set of prime numbers. Let truth set $T = \{x \in D \mid S(x)\}$. Please show the elements inside the set T .

Problem 6 Proof: 10pts

We showed in class that $p \rightarrow q$ does not necessary also mean $q \rightarrow p$.

Which of the following statement is correct $\forall x \in \mathbb{Z}$ that (a) if $x = 1$ then $x^2 = 1$ (b) if $x^2 = 1$ then $x = 1$? If you agree, please prove, if you do not agree, please provide counter example to disprove.

Problem 7 Mersenne Number: 5+10 pts

We talked about Mersenne Number and Mersenne Prime Number in class.

(a) List 2 applications of prime numbers in computer science

(b) Given a Mersenne prime $2^n - 1$, please show that $2^{n-1}(2^n - 1)$ is a perfect number.

Problem 8 Proof: 5+ 5+ 5 pts

Prove or disprove. If the statement is correct, please prove it; otherwise, please disprove by an counter example.

(a) The sum of three consecutive integers is zero (mod 3).

(b) The prodcut of two even integers is zero (mode 4)

. (c) For all odd integers n , $3n + 3$ is divisible by 6.

Problem 9 Propositional Logic: 5pts

Is $(p \wedge \sim q) \wedge (\sim p \vee q) \wedge r$ a tautology, contradiction or neither? Please use algebraic rules.

Problem 10 Floor, Modulo, Ceiling functions: 5 * 3 pts

Please compute the following expression

(a) $\lceil (((23 \% \lceil 5.23 \rceil) \times \lfloor 3.23 \rfloor) / \lceil 3.23 \rceil) \rceil$

(b) $(\lfloor -3.4 \rfloor * \lceil 5.3 \rceil) + \lceil (2.5 * 3.3) \rceil$

(c) $\lceil (((23 + \lceil -5.23 \rceil) \% \lfloor 3.23 \rfloor) / \lceil 3.23 \rceil) \rceil$

Problem 11 Predicate Logic: 5 + 5: 10pts

(a) Why the statement $\forall x \in D, (P(x) \vee Q(x))$ is not the same as $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$? Given an example.

(b) Why the statement $\exists x \in D, (P(x) \wedge Q(x))$ is not the same as $(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$? Given an example.

Problem 12 Proof: 10pts

Let $X = 1 + (2^1 + 3^1) + (2^2 + 3^2) + \cdots + (2^k + 3^k)$. Please find the closed form for X .