MAT 115: Finite Math for Computer Science
Problem Set 3

Due: 03/29/2019

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/147**

Problem 1 Proof: Combinatorial 15: 5 + 10 pts

We talked about the combination. $C(n,k)$ means how many ways one can pick k distinct items out of n distinct items (grab k items at one time out of n items). It is know that $C(n,k) = \frac{n!}{(n-k)!k!}$ where $T! = 1 \times 2 \times 3 \times \cdots \times T$ for any positive integer T. Please show

(a) $\frac{n!}{(n-k)!k!} = (n \times (n-1) \times \cdots \times (n-k+1))/k!$

(b) Please show that $C(n-1,k-1) + C(n-1,k) = C(n,k)$

Problem 2 Permutation: No Repetition 2*7 = 14

We work with the ordinary alphabet of **28-letters** (A-Z plus τ, λ. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)

Problem 3 Permutation: (3+3+9=15 pts)

We are interested in forming 3 letter words using the letters in uticaalbanysyracuse. For the purpose of the problem, a word is any **list** of letters.
(a) How many words can be made with no repeated letters?

(b) How many words can be made with unlimited repetition allowed?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in uticaalbanysyracuse?

Problem 4 Permutation + Combinatorics: (20 pts)

Please explain how many ways to put balls into bins based on the condition:
(a) 4 identical bins and 4 identical balls

(b) 4 different bins (A, B, C, D) and 4 identical balls

(c) 4 identical bins and 4 different balls (Red, White, Green, Purple)

(d) 4 different bin (A, B, C, D) and 4 different balls (Red, White, Green, Purple)

Problem 5 Permutation: With and Without Repetition (5×3 = 15 pts)

We are interested in forming 3 letter words using the letters in THELITTLESTGUY.
(a) How many words can be made with no repeated letters?

(b) How many words can be made with unlimited repetition allowed?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in THELITTLESTGUY?

Problem 6 Probability + Counting (3× 3 × 2 = 18 pts)

An urn A contains ten labeled balls while each label contains a number, ranging from 1,2, $\cdots$ to 10. An urn B contains five labeled balls while the number is ranging from 1,2, $\cdots$ to 5.
(a) Two balls are drawn, one from A and one from B. What is the sample space? What is the probability that the sum of the labels on the balls is odd? What is the probability that the sum of the labels on the balls is 9?

(b) Two balls are drawn one after the other without replacement and the order matters from urn A. What is the sample space? What is the probability that the sum of the labels on the balls is odd and the first ball number must be greater than the 2nd ball number)? What is the probability that the sum of the labels on the balls is 9?

(c) Two balls are drawn from urn B one after the other with replacement and the order matters. What is the sample space? What is the probability that the sum of the labels on the balls is even? What is the probability that the sum of the labels on the balls is 10?

Problem 7 Permutations: 20 points

We have 7 kids to sit in a rowl. Let say the boys are A, B, C, D and the girls are E, F and G. Please compute the number of ways to seat the kids based on the following constraints
(a) Each girl must sit between boys

(b) C does not sit next to E and A does not sit next to G and B and F must sit together

(c) A must sit on the right of B, and E must sit at least 2 seats away from G (i.e. at least one 1 seat in between)

(d) B must sit next to C and D must also sit next to C and A does not want to sit next to G

Problem 8 Probability: 20 pts

We are tossing 10 biased coins at the same time. The coins have the probability of 0.4 to get Head and 0.6 to get Tail.
(a) What is the probability of getting 10 Heads?

(b) What is the probabiliyt of getting 4 Heads and 6 Tails?

(c) What is the probability of getting more Tails then Heads

(d) What is the probability of getting even numbers of Tails

(e) What is the probability of getting odd numbers of Tails? Does your answer here along with answer in (d) sum up to 1?

Problem 9 Induction Proof: 10 points

Please show that $\sum_{i=1}^{n} i^2 = \frac{(n)(n+1)(2n+1)}{6}$
Base case:

Hypothesis

Induction