MAT 115: Finite Math for Computer Science
Problem Set 4

Due: 04/20/2019

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/105**

Problem 1 Sets: Linear Order: 5 + 5 pts

Let $A = \{1, 5, 4\}$, $B = \{u, v\}$ and $C = \{t, s\}$. Take the linear orders on A to be numeric and alphabetical orders on B and C. List the elments in each of the following sets in lexicographic order.
(a) A x (B x C) (use lex order on B x C)

(b) (A x B) X C (juse lex order on A x B)

Problem 2 Set: subset proof: 5 + 5 pts

By using the definition of subset, please show that
(a) If A , B, and C are subsets of U, then $A \subseteq B$ and $A \subseteq C$ implies that $A \subseteq (B \cap C)$.

(b) If A , B, and C are subsets of U, thne $A \subseteq C$ and $B \subseteq C$ implies that $(A \cup B) \subseteq C$.

Problem 3 Power set: 5 + 5 + 5 pts

Given a set $A = \{a, b, t\}, B = \{g, k, i, a, t\}$, please compute the following:
(a) $A \times A$

(b) $B \times A$

(c) Power set of $(B - (B \cap A))$

Problem 4 Relation & Partition: 10 + 5 + 5 pts

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1), (2, 3), (3, 2)\}$.
(a) Is R an equivalence relation? (need to verify those three properties)

(b) What are the equivalent classes (partitions) of A

(c) $A = \{1, 2, 3, 4, 5, 6\}$ and it can be partitioned by R_2 such that the partitions are $P_1 = \{1, 5\}, P_2 = \{2, 4, 6\}$ and $P_3 = \{3\}$. Please write out the relation R_2.

Problem 5 Relation: 5+5 pts

Let $A = \{1, 5\}$ and $B = \{\alpha, \beta, \delta\}$. Let R be a relation that is a subset of $A \times B$.
(a) How many relations are there when $|R| = 3$? And please given on instance of R.

(b) How many relations are there when $|R| = 6$? And please given on instance of R.

Problem 6 Power set : 5 + 5 pts

Compare the following paris of sets. Can they be equal? Is one a subset of the other? Can they have the same size?
(a) $\mathcal{P}(A \cup B)$ and $\mathcal{P}(A) \cup \mathcal{P}(B)$

(b) $\mathcal{P}(A \times B)$ and $\mathcal{P}(A) \times \mathcal{P}(B)$

Problem 7 Equivalence classes: 10 pts

Given a binary relation R being an equivalence relation acting on set A, i.e. $R \subseteq A \times A$, the equivalence classes of R are defined as

$$S_x = \{z|(x,z) \in R\}$$

Please show that $\forall x, y \in A, x \neq y$, we have $S_x \cap S_y = \phi$ or $S_x = S_y$.

Problem 8 Set algebraic rules: 5 + 5 pts

(a) The following is false for subsets of a set U. Please draw Venn diagram to represnt the situation being described. For all sets, A, B, and C $(A-B)-C = A-(B-C)$.

(b) Use the algebraic rules to show the following: [Hint: $D - E = D \cap \bar{E}$]
If A and B are subsets of U, then $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Problem 9 Relation: Equivalence 10pts

Define integers $x \equiv y$ to be related if $d|(x-y)$. Show that $\equiv$ is an equivalence relation by defining a function M that xMy when $d|(x-y)$.

Reflexive:

Symmetric:

Transitive: