MAT 115: Finite Math for Computer Science
Problem Set 5

Due: 05/03/2019

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: /90

Problem 1 Function: 5* 3 = 15pts

Let $S = \{f | f : A \to B$, f is injective $\}$. Let us assumes the elements in each set are different. For the following cases, find $|S|$
(a) $|A| = 3, |B| = 2$

(a) $|A| = 3, |B| = 5$

(a) $|A| = m, |B| = n, m < n$

Problem 2 Types of Functions: I: 10*2 = 20pts

Let A and B be finite sets and $f : A \to B$. Prove/disprove the following:
(a) If f is injective, then $|A| \leq |B|$. [Hint: easier with contrapositive proof]

(b) If $|A| \leq |B|$, then f is injective. [Hint: Maybe you can disprove by example]

Problem 3 Types of Functions: II: 10*2 = 20pts

Let A and B be finite sets and $f : A \to B$. Prove/disprove the following:
(a) If f is surjective, then $|A| \geq |B|$.

(b) If $|A| \geq |B|$, then f is surjective.

Problem 4 Equivalence Relation: 5* 3 = 15pts

Let $\mathbb{Z}$ be the set of integers. Define a relation R where $(x, y) \in R$ and $x, y \in \mathbb{Z}$ that 5 divides $x^2 - y^2$. Please show R is an equivalence relation.

(a) Reflexive

(b) Symmetric

(c) Transitive

Problem 5 Pigeon Holes: 5pts

There are N students in the class. Their exam score ranges between 32 and 99. All possible scores are achieved by students except 45, 67, 55, 37. What is the smallest N that guarantees that at least three students achieved the same score?

Problem 6 Pigeon Holes: 15pts

Let A $= \{t_1, \cdots, t_n\}$ be a set of n **different** integers. For instance A=$\{t_1, t_2, t_3, t_4\} = \{1, 3, 6, 9\}$ is a set of 4 different numbers. Show that either n divides t_k for some $t_k \in A$ or n divides $(t_i - t_j)$ for $t_i, t_j \in A$ and $i \neq j$.