MAT 115: Exam 1

Section: MWF 9:20-10:30 am

Date: 02/25/2019

Instructions:
You have **70** minutes for this exam. There are 9 problems. **Calculators are allowed**. The total score is **90** plus extra 30 points. That is this exam set has **120** points and perfect score is 90. If you get 120, that means you have $\frac{400}{3} \simeq \mathbf{133}$ out of 100 for exam 1. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use your time and space wisely.

First Name:

Last Name:

Score: **/90**

Problem 1 Truth Table + Circuit design (10 + 5pts)

(a) Make a truth table for $f = ((\sim p \vee \sim r) \wedge (\sim q \oplus q)) \vee (\sim p \wedge (q \vee r))$

p	q	r	**f**
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

(b) Construct a combinatorial circuit using NOT gates, OR gates and AND gates that produces the output as f does from input bits p, q, and r. Please make sure your circuit is **as short as possible**.

Problem 2 Base Change : 15 (5+5+5)pts

(a) Please convert 2019_{10} into a base 16 number. You must show the computation

(b) Please convert $ABBA_{16}$ into a base 10 number.

(c) Please show the result of $1234_{10} + ABBA_{16}$ in base 13.

Problem 3 Algebraic Rules+Boolean Functions (10 + 5 pts)

(a) Let p, q and r be boolean variables and let $f_1 := ((\sim p \vee q) \wedge (p \vee \sim r)) \wedge (\sim p \vee \sim q)$ and function $f_2 :=\sim (p \vee r)$. Is f_1 equal to f_2? If yes, please **prove** it. If not, please simplify f_1 to its simplest form. Please use algebraic rules. If you are stuck, you can try truth table but you would only get half of the credits if your prove by truth table.

(b) Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology (always evaluates to truth) by using algebraic rules (proposition equivalence rules)

Problem 4 Logic: Quantifiers (5+5+5 pts)

Is the following statement true or false? Explain why. If you disprove, you must provide the counter example.
(a) $\exists x \in D, (P(x) \wedge Q(x))$ is the same as $(\exists x \in D, P(X)) \wedge (\exists x \in D, Q(x))$

Let $Q(x)$ be the statement, given $x, y, z \in \mathbb{Z}$ and $x \neq 0$. what are the truth value of the following expressions? If true, find the corresponding values.

(a) $\exists x \exists y \; Q(x, y) := (x + y)/x = (x - y)$,

(b) $\exists Z \forall x \forall y \; Q(x, y) := ((x + y)/x)z = (x - y)z$,

Problem 5 Predicate Logic (10pts)

Let $D = \{1, 3, 4, 5, 9, 49, 64, 81\}$, $S(x) = (\sqrt{x} \in \mathbb{Z} \wedge ((\sqrt{x} - 1) \in \mathbb{P} \vee (\sqrt{x} + 2) \in \mathbb{P})) \vee (\sqrt{x+3} \in \mathbb{Z} \wedge (\sqrt{x+3})\%2 = 1)$. Let $T = \{x \in D | S(x)\}$. Please show the elements inside the set T.

Problem 6 Floor, Modulo, Ceiling functions: 5 * 3 pts

Please compute the following expression
(a) $\lceil(((22\%\lceil 6\rceil) \times \lfloor 4.23\rfloor)/\lceil 5.23\rceil)\rceil$

(b) $(\lfloor -5.4\rfloor * \lceil 4.3\rceil) + \lceil(2.7 * 3.3)\rceil$

(c) $\lfloor(((23 + \lceil 5.23\rceil)\%\lceil 4.23\rceil)/\lceil 2.23\rceil)\rfloor$

Problem 7 Proof: 10pts

For all **odd** integers n and m, please show $k = (3n+3)(m^2 - n^2)$ is divisible by 12. [hint: 12 = 3*2*2, if you can show you can find 3 and $4 = 2 * 2$ as the factors for k, then you finish the proof]

Problem 8 Proof: 5 + (3+2+5) = 15pts

(b) Let $X = 1 + (4^1 - 3^1) + (4^2 - 3^2) + \cdots + (4^k - 3^k)$. Please find the closed form for X; then prove your result via induction proof.
(i) Closed form:

(II) Base case:

(III) Hypothesis:

(IV) Induction:

Problem 9 Mersenne Prime: 2+8 pts

(a) What is a Mersenne Prime and what is a perfect number?

(b) Show that $N = 2^{n-1}(2^n - 1)$ is a perfect number when $2^n - 1$ is a Mersenne prime number.

Problem 10 Algebraic Rules Sheet

Theorem 2 (Algebraic rules for Boolean functions) *Each rule states that two different-looking Boolean functions are equal. That is, they look different but have the same table.*

Associative Rules:	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$	$(p \vee q) \vee r = p \vee (q \vee r)$
Distributive Rules:	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
Idempotent Rules:	$p \wedge p = p$	$p \vee p = p$
Double Negation:	$\sim\sim p = p$	
DeMorgan's Rules:	$\sim(p \wedge q) = \sim p \vee \sim q$	$\sim(p \vee q) = \sim p \wedge \sim q$
Commutative Rules:	$p \wedge q = q \wedge p$	$p \vee q = q \vee p$
Absorption Rules:	$p \vee (p \wedge q) = p$	$p \wedge (p \vee q) = p$
Bound Rules:	$p \wedge 0 = 0 \qquad p \wedge 1 = p$	$p \vee 1 = 1 \qquad p \vee 0 = p$
Negation Rules:	$p \wedge (\sim p) = 0$	$p \vee (\sim p) = 1$

Problem 11 Scratch Paper

Do not detach the paper.

Problem 12 Scratch Paper

Do not detach the paper.