MAT 115: Exam 2

Section: MWF 9:20-10:30 am

Date: 04/05/2019

Instructions:
You have **80** minutes for this exam. There are 8 problems. **Calculators are allowed**. The total score is **110** plus extra 20 points. That is this exam set has **130** points and perfect score is 110. If you get 130, that means you have $\frac{1300}{11} \simeq \mathbf{120}$ out of 100 for exam 2. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use your time and space wisely.

First Name:

Last Name:

Score: **/130**

Problem 1 Permutation: No Repetition 5*4 = 20

We work with the ordinary alphabet of **26-letters** (A-Z. Please solve the following: (no repetition allowed)
(a) Define a 6-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 6-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 2 vowels (one of them has to be an O)

(b-2) How many 5-letter words with exactly 3 vowels (one of them has to be a U)

(b-3) How many 5-letter words with exactly 5 vowels

Problem 2 Proof: Combinatorial 15: 8 + 10 pts

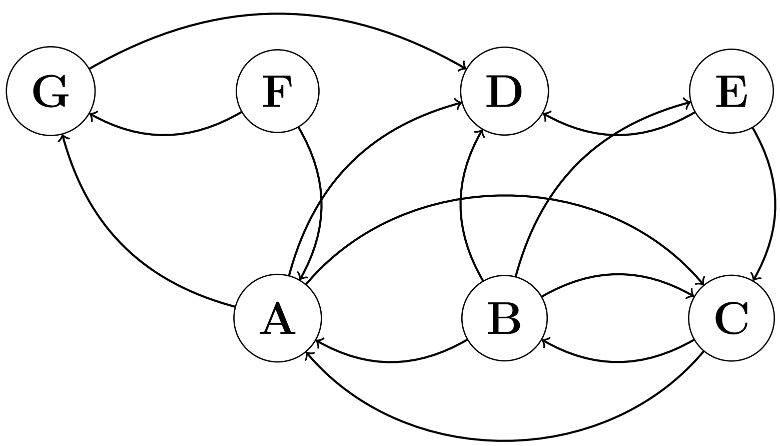
(a) Please show that $C(n-1,k-1)+C(n-1,k)=C(n,k)$

(b) $S(n,k)$ is the Sterling number of the 2nd kind; that is to toss n distinct objects into exactly k bins. We know that $S(n,k)=S(n-1,k-1)+kS(n-1,k)$. Please find the close form for $S(n,2)$[hint: draw the tree].

Problem 3 Graph: 5 + 5 + 5 pts

For a graph $G = (V, E)$, let $d(v)$ be the degree of the vertices $v \in V$.
(a) A complete **directed** graph $G = (V, E)$ is a graph where each node v_i in V has one edge to each other nodes in V. Given $|V| = n$, that means $d(v_i) = n - 1, \forall v_i \in V$. What is $|E|$?

(b) Suppose you are given the following **directed** graph $G = (V, E)$. Please find Cycle 1 contains 3 distinct vertices.



(c) Is is possible to find a cycle of more than 3 distinct vertices? If not possible, what absorbing/radiating nodes are causing the problem? And what nodes are affected by those ?

Problem 4 Permutation: (5*3=15 pts)

We are interested in forming 3 letter words using the letters in mississippi. For the purpose of the problem, a word is any **list** of letters.

(a) How many words can be made with no repeated letters?

(b) How many words can be made if repeats are allowed but no letter can be used more than it appears in mississippi?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in mississippi and ‘s’ must appear at least once?

Problem 5 Permutation + Combinatorics: (5*4 = 20 pts)

Please explain how many ways to put balls into bins based on the condition:
(a) 3 identical bins and 3 identical balls

(b) 3 different bins (A, B, C) and 3 identical balls

(c) 3 identical bins and 3 different balls (Red, White, Green)

(d) 3 different bin (A, B, C) and 4 different balls (Red, White, Green)

Problem 6 Probability + Counting ((1+2+3)*2 = 12 pts)

Urn A contains labeled balls while each label contains a number, ranging from 1,2, $\cdots$ 8. Urn B contains five labeled balls while the number is ranging from 1,2, $\cdots$ 4.
(a) Three balls are drawn, two from A (one after another) and one from B. What is the sample space? What is the probability that the drawn balls are (in sequence) even, odd then even? What is the probability that the sum of the labels on the balls is 9?

(b) Three balls are drawn one after the other with replacement and the order matters from urn A. What is the sample space? What is the probability that the sum of the labels on the balls is odd and the first ball number must be greater than the 2nd ball number)? What is the probability that the sum of the labels on the balls is 9?

Problem 7 Permutations: 5*4 = 20 points

We have 8 kids to sit in a rowl. Let say the boys are A, B, C, D, E and the girls are F, G and H. Please compute the number of ways to seat the kids based on the following constraints

(a) Each girl must sit between boys [muts choose 3 spots for girls first]

(b) C does not sit next to E and A does not sit next to G and B and F must sit together

(c) A must sit on the right of B, and E must sit at "exactly" 2 seats away from G (i.e. 1 seat in between)

(d) B must sit next to C, D must also sit next to C, H must sit next to D and A does not want to sit next to G

Problem 8 Induction Proof: 10 points

Please show that $\sum_{i=1}^{n}(2 * i^2 + 1) = \frac{(n)(n+1)(2n+1)+3n}{3}$
Base case:

Hypothesis

Induction

Problem 9 Scratch Paper

Do not detach the paper.