MAT 115: Exam 3

Section: MWF 9:20-10:30 am

Date: 05/01/2019

Instructions:
You have **100** minutes for this exam. There are 8 + 2 problems. **Calculators are allowed**. The total score is **100** plus extra 20 points. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. Set algebraic rules are also attached at the end of this exam. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use your time and space wisely.

First Name:

Last Name:

Score: **/100 +** **/20**

Problem 1 Relation & Partition: (3*3) + 3 + 3 pts

Let $A = \{1, 2, 3, 4, 5\}$ and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (4, 3), (3, 4), (5, 5)\}$.
(a) Is R an equivalence relation? (need to verify those three properties)

(b) What are the equivalent classes (partitions) of A

(c) $A = \{1, 2, 3, 4\}$ and it can be partitioned by R_2 such that the equivalence classes are $S_1 = \{1, 4\}, S_2 = \{2\}$ and $S_3 = \{3\}$. Please write out the relation R_2.

Problem 2 Function: 5* 3 = 15pts

Let $S = \{f | f : A \to B$, f is injective $\}$ and $T = \{g | g : B \to C$, g is bijective $\}$. Let $A = \{a_1, a_2, \cdots, a_p\}, B = \{b_1, b_2, \cdots, b_q\}, C = \{c_1, c_2, \cdots, c_r\}$. Let us assumes the elements in each set are different. For the following cases, find $|S| \times |T|$

(a) $p = 5, q = 4, r = 4$

(a) $p = 4, q = 5, r = 5$

(a) $p = m, q = r = n$ and $m \leq n$ Please express the result in terms of m, n.

Problem 3 Lexigraphical Order + Power set: 5 + 5 pts

With $A = \{b, a\}, B = \{g, a, t\}, C = \{b\}$, compute the following in lexigraphical order:
(a) $(B \times A) \times C$

(b) Power set of $((B - (B \cap A)) \cup C)$

Problem 4 Power set : 5 + 5 pts

Compare the following paris of sets. If not equal, please explain why.
(a) $\mathcal{P}(A \cap B)$ and $\mathcal{P}(A) \cap \mathcal{P}(B)$

(b) $\mathcal{P}(A \cup B)$ and $\mathcal{P}(A) \cup \mathcal{P}(B)$

Problem 5 Types of Functions: 10*2 = 20pts

Let A and B be finite sets and $f : A \to B$. Prove/disprove the following:
(a) If f is injective, then $|A| \leq |B|$. [Hint: easier with contrapositive proof]

(b) If f is surjective, then $|A| \geq |B|$.

Problem 6 Pigeon Holes: 5pts

There are N students in the class. Their exam score ranges between 55 and 99. All possible scores are achieved by students except 78, 67, 55. What is the smallest N that guarantees that at least three students achieved the same score?

Problem 7 Set algebraic rules: 5 + 5 pts

(a) The following is false for subsets of a set U. Please draw Venn diagram to represnt the situation being described. For all sets, A, B, and C that $(A \cup B) \cap C = A \cup (B \cap C)$.

(b) Use the algebraic rules to show the following: [Hint: $D - E = D \cap \bar{E}$]
If A and B are subsets of U, then $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Problem 8 Relation: Equivalence 15pts

Let $x, y \in \mathbb{Z}$ and R be a relation such that $(x, y) \in R$ when $7|[(x^3 - y^3) + (x^2 - y^2)]$. Show that R is an equivalence relation.

Reflexive:

Symmetric:

Transitive:

Problem 9 Bonus: Pigeon Holes: 10pts

Let A = $\{t_1, \cdots, t_n\}$ be a set of n **different** integers. For instance A=$\{t_1, t_2, t_3, t_4\} = \{1, 3, 6, 9\}$ is a set of 4 different numbers. Show that either n divides t_k for some $t_k \in A$ or n divides $(t_i - t_j)$ for some $t_i, t_j \in A$ and $i \neq j$.

Problem 10 Bonus: Equivalence classes: 10 pts

Given a binary relation R being an equivalence relation acting on set A, i.e. $R \subseteq A \times A$, the equivalence classes of R are defined as

$$S_x = \{z|(x,z) \in R\}$$

Please show that $\forall x, y \in A, x \neq y$, we have $S_x \cap S_y = \phi$ or $S_x = S_y$.

Problem 11 Scratch Paper

Do not detach the paper.

Problem 12 Scratch Paper

OK to detach the paper.

Problem 13 Set Algebraic Rules Sheet

Associative:	$(P \cap Q) \cap R = P \cap (Q \cap R)$	$(P \cup Q) \cup R = P \cup (Q \cup R)$
Distributive:	$P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$	$P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$
Idempotent:	$P \cap P = P$	$P \cup P = P$
Double Negation:	$\sim\sim P = P$	
DeMorgan:	$\sim(P \cap Q) = \sim P \cup \sim Q$	$\sim(P \cup Q) = \sim P \cap \sim Q$
Absorption:	$P \cup (P \cap Q) = P$	$P \cap (P \cup Q) = P$
Commutative:	$P \cap Q = Q \cap P$	$P \cup Q = Q \cup P$