MAT 115: Final Exam

Section: MWF 9:20-10:30 am

Date: 05/08/2019

Instructions:
You have **120** minutes for this exam. There are 10 problems. **Calculators are allowed**. The total score is **125**. Extra blank sheets are also attached at the end of this exam if you need more computation space. Algebraic rules are provided in the back. Please use your time and space wisely. Be reminded, parameters for the problems have been **changed** (i.e. different from exam 1 - 3), please read the problems carefully.

First Name:

Last Name:

Score: /125

Problem 1 Truth Table + Circuit design (10 + 5pts)

(a) Make a truth table for $f = ((p \vee r) \wedge (\sim q \oplus q)) \vee (\sim q \wedge (p \vee r))$

p	q	r	**f**
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

(b) Construct a combinatorial circuit using NOT gates, OR gates and AND gates that produces the output as f does from input bits p, q, and r. Please make sure your circuit is **as short as possible**.

Problem 2 Base Change : 5 + 5 pts

(a) Please convert 2019_{10} into a base 7 number. You must show the computation. Answer without computation process will not be given credits.

(b) Please convert $ABBA_{13}$ into a base 10 number. You must show the computation. Answer without computation process will not be given credits.

Problem 3 Algebraic Rules (10 pts)

Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology (always evaluates to truth) by using **algebraic rules**. Proof by truth table can obtain up to up most half of the credits.

Problem 4 Floor, Modulo, Ceiling functions: 5 * 3 pts

Please compute the following expression. Answer without computation process will not be given credits.

(a) $\lceil (((22\%\lceil 7\rceil) \times \lfloor 4.23\rfloor)/\lceil 5.23\rceil)\rceil$

(b) $(\lfloor -5.4\rfloor * \lceil 5.3\rceil) + \lceil (2.7 * 3.6)\rceil$

(c) $\lfloor (((23 + \lceil 5.8\rceil)\%\lceil 4.73\rceil)/\lceil 2.23\rceil)\rfloor$

Problem 5 Permutation + Combinatorics: (5*3 = 15 pts)

Please explain how many ways to put balls into bins based on the conditions. Wrong answers (simply throwing a number there) without computation process (analysis) will not be given credits.

(a) 3 different bins (A, B, C) and 4 identical balls

(b) 3 identical bins and 4 different balls (Red, White, Green, Purple)

(c) 3 different bin (A, B, C) and 4 different balls (Red, White, Green, Purple)

Problem 6 Permutations: 5*3 = 15 points

We have 7 kids to sit in a row. Let say the boys are A, B, C, D and the girls are E, F and G. Please compute the number of ways to seat the kids based on the following. Wrong answers (simply throwing a number there without any analysis process/explanation) will not be given credits.
(a) Each girl must sit between boys

(b) C does not sit next to E and A does not sit next to G and B and F must sit together

(c) A must sit on the right of B, and E must sit at “exactly” 3 seats away from G (i.e. 2 seats in between)

Problem 7 Induction Proof: 10 points

Please show that $\sum_{i=1}^{n}(i^2) = \frac{(n)(n+1)(2n+1)}{6}$. Please provide the computation process.
Base case:

Hypothesis

Induction

Problem 8 Function: 5* 3 = 15pts

Let $S = \{f|f : A \to B$, f is injective $\}$ and $T = \{g|g : B \to C$, g is bijective $\}$. Let $A = \{a_1, a_2, \cdots, a_p\}, B = \{b_1, b_2, \cdots, b_q\}, C = \{c_1, c_2, \cdots, c_r\}$. Let us assumes the elements in each set are different. For the following cases, find $|S| \times |T|$. Wrong answers (simply throwing a number there) without computation process (analysis) will not be given credits.

(a) $p = 5, q = 5, r = 4$

(a) $p = 4, q = 6, r = 3$

(a) $p = q = r = m$. Please express the result in terms of m.

Problem 9 Power set : 5 + 5 pts

Let $A = \{2, 4, 5, 6\}, B = \{1, 3, 4\}$. Compute the power set. Please show the computation process.
(a) Compute $\mathcal{P}(A) \cap \mathcal{P}(B)$

(b) Compute $\mathcal{P}(A - (A \cap B))$

Problem 10 Set algebraic rules: 5 + 5 pts

(a) The following is **false** for subsets of a set U. Please draw Venn diagram to represnt the situation being described. For all sets, A, B, and C that $(A \cap B) \cup C = A \cap (B \cup C)$.

(b) Use the algebraic rules to show the following: [Hint: $D - E = D \cap \bar{E}$]
If A and B are subsets of U, then $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$. Please show the computation process.

Problem 11 Algebraic Rules Sheet

Theorem 2 (Algebraic rules for Boolean functions) *Each rule states that two different-looking Boolean functions are equal. That is, they look different but have the same table.*

Associative Rules:	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$	$(p \vee q) \vee r = p \vee (q \vee r)$
Distributive Rules:	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
Idempotent Rules:	$p \wedge p = p$	$p \vee p = p$
Double Negation:	$\sim\sim p = p$	
DeMorgan's Rules:	$\sim(p \wedge q) = \sim p \vee \sim q$	$\sim(p \vee q) = \sim p \wedge \sim q$
Commutative Rules:	$p \wedge q = q \wedge p$	$p \vee q = q \vee p$
Absorption Rules:	$p \vee (p \wedge q) = p$	$p \wedge (p \vee q) = p$
Bound Rules:	$p \wedge 0 = 0 \qquad p \wedge 1 = p$	$p \vee 1 = 1 \qquad p \vee 0 = p$
Negation Rules:	$p \wedge (\sim p) = 0$	$p \vee (\sim p) = 1$

Problem 12 Set Algebraic Rules Sheet

Associative:	$(P \cap Q) \cap R = P \cap (Q \cap R)$	$(P \cup Q) \cup R = P \cup (Q \cup R)$
Distributive:	$P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$	$P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$
Idempotent:	$P \cap P = P$	$P \cup P = P$
Double Negation:	$\sim\sim P = P$	
DeMorgan:	$\sim(P \cap Q) = \sim P \cup \sim Q$	$\sim(P \cup Q) = \sim P \cap \sim Q$
Absorption:	$P \cup (P \cap Q) = P$	$P \cap (P \cup Q) = P$
Commutative:	$P \cap Q = Q \cap P$	$P \cup Q = Q \cup P$

Problem 13 Scratch Paper

Do not detach the paper.

Problem 14 Scratch Paper

Do not detach the paper.

Problem 15 Scratch Paper

Do not detach the paper.