MAT 115: Finite Math for Computer Science
Problem Set 1

Out: 09/12/2017 Due: 09/21/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:
Last Name:
Group ID:
Score:     /90
Problem 1  Truth Table: 10pts

Make a truth table for \((\sim p \land (\sim p \lor q)) \land \sim (q \lor \sim r)\)
Problem 2 Proof: Algebraic Rules: 10pts

Is the function \((r \lor p) \land (\sim r \lor (p \land q)) \land (r \lor q)\) equal to the function \(p \land q\)
Problem 3 Boolean Functions: 15pts

Given a function $f : \{0, 1\}^4 \rightarrow \{0, 1\}$, please answer the following:

(a) Please show two input instances from the domain.

(b) What is the number of possible boolean functions $f$?

(c) Boolean functions can be found in many applied problems. Please briefly describe a problem (search on wikipedia) that is a direct application of boolean function and explain why this problem is important.
Problem 4  Base Change: 20pts

Convert the following numbers
(a) ED0A (hex number into decimal form)

(b) 1201211 ((ternary number into decimal form)

(c) 2017 (base 8 number to decimal number)

(d) 115 (decimal number into ternary form (base 3))
Problem 5  Representing Function: 10pts

Given $f : \{0, 1\}^3 \to \{0, 1\}$, we can easily interpret it as $f(p, q, r) = s$ where $p, q, r \in \{0, 1\}$ and $s \in \{0, 1\}$. If we have $f(0, 0, 0) = 1$, $f(0, 1, 0) = 1$, $f(1, 0, 0) = 0$, $f(1, 1, 0) = 0$, $f(0, 0, 1) = 1$, $f(0, 1, 1) = 1$, $f(1, 0, 1) = 0$ and $f(1, 1, 1) = 1$. Please derive the boolean function $f$. 
Problem 6  Circuits: 10+5pts

For the addition circuit on page BF-19, the operator for generating C is an OR gate. Normally, with intuition, it should be another half adder to add up $c'$ and $c''$.

(a) What is the reason that we can use OR gate there to add up $c'$ and $c''$?

(b) Furthermore, can we use XOR to replace that OR?
Problem 7  Circuits: 10pts

Design a circuit that represent the Boolean function $S$ where $S(P, Q, R) = 0$ if only if $(P, Q, R) = (0, 0, 0)$ or $(P, Q, R) = (1, 1, 1)$
Problem 8  Practice Problems

For practice only. You do not have to turn in the solution.
Unit BF: 1.12, 1.14, 1.15, 2.1, 2.4, 2.5, 2.6