Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: /115 + /10 (bonus)
Problem 1  Simple Induction: 2+3+5 pts

Please use induction proof method to show that $\sum_{i=1}^{n} i$ has a closed form as $\frac{n(n+1)}{2}$

Base case:

Hypothesis:

Induction:
Problem 2  Euclidean algorithm: 5+5 pts

Use the Euclidean algorithm to find all common divisors of

(a) 1001 and 544

(b) 252 and 180
Problem 3  Euler function: 5+5+5 pts

Use the Euler number for the following number

(a) $N = 2^4 \times 5 \times 7$

(b) $N = 4 \times 10 \times 15$

(c) $N = 21$
Problem 4  Use of Subtraction Trick: 20 pts

Prove each of the following identities from the basic algebraic rules for sets. You may want to use the fact that $D - E = D \cap \sim E$ [Here the $\sim$ means the complement].

(a) If $A$ and $B$ are subsets of $U$, then $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

(b) If $A$, $B$, and $C$ are subsets of $U$, then $(A - B) - C = (A - C) - B$
Problem 5  Disprove via Venn Diagram: 5+5 pts

Show the following statement is wrong via Ven Diagram example.
(a) For all sets $A$, $B$ and $C$, we have $(A \cup B) \cap C = A \cup (B \cap C)$

(b) If $A$, $B$, and $C$ are subsets of $U$, then $(A - B) - C = A - (B - C)$
Problem 6  Ordered Set: 5+5 pts

Let $A = w, x, y$ and $B = a, b, c$. List the elements in each of the following sets in lexicographic order:
(a) $B \times A$

(b) $B \times (A \times B)$
Problem 7  Combinations: 5+5 pts

Compute
(a) $C(14, 5)$

(b) $C(23, 6)$
Problem 8  Power Set: 5 + 5 + 5 + 5 pts

$A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, compute the following:

(a) Power set of $A - B$.

(b) Power set of $B - A$.

(c) Let $C = A \cup B$. What is $|C|$?

(d) $P(P(P(\{1, 2\}))$ here $P$ means power set.
Problem 9 Partition: 10 pts

How many refinements are there of the partition \( \{\{1, 3, 5\}, \{2, 6\}, \{4, 7, 8, 9\}\} \)? Please explain.
Problem 10  Bonus: 10 pts

Let $S = \{1, 2, \cdots, n\}$. Let $S_1$ be the set of all subsets of that that contain 1. Let $T_1$ be the set of all subsets of $S$ that do not contain 1. Show that $T_1 = S_1 = 2^{n-1}$. 
Problem 11 Practice Problems

For practice only. You do not have to turn in the solution.
Unit SF: 1.6, 1.8, 1.9, 1.14, 2.19