CS 528: Quantum Computation
Assessment Exam

MW: 12:00 - 1:15 pm

10/06/2019

Instructions:
I leave plenty of space on each page for your computation. You have 75 minutes for this test and the total score is 75 + 10 (bonus). Please use your time wisely.

First Names:

Group ID:

Score: /75 + 10
Problem 1  Formula : 10 pts [Bonus]

Please show that $H^\otimes_n |j\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} (-1)^{ij} |i\rangle$ via induction. Ther term $ij$ is the inner product of vectors $|i\rangle$ and $|j\rangle$ (they are both $\in \mathbb{R}^n$).
Problem 2  Qiskit + Deutsch Jozsa : 10 + 5 pts

(a) Please write out the Qiskit source code implementing the Deutsch-Jozsa
(b) Please show the circuit implementing the Deutsch-Jozza
Problem 3  Grover : 10 + 10 pts

(1) Please mark the area of circuit that is performing the 2nd reflection $2|\psi\rangle\langle\psi| - I$[2pts]. And what is missing [need to justify] and why this missing part will not affect the Grover search result[8pts]?
(2) Please show that for a generic Grover, assuming the initial state

\[ |\psi\rangle = \frac{1}{\sqrt{N}} \sum_i |i\rangle = \sin \theta |G\rangle + \cos \theta |B\rangle \]

Please derive and verify the G operator in the $|G\rangle, |B\rangle$ basis.
Problem 4  Simon’s algorithm: 5+5+5 pts

Suppose we run Simon’s algorithm on the following input $x$ (with $N = 8$, and hence $n = 3$):

\[
\begin{align*}
    x_{000} &= x_{101} = 000 &    x_{001} &= x_{100} = 001 \\
    x_{010} &= x_{111} = 010 &    x_{011} &= x_{110} = 011
\end{align*}
\]

Note that $s$ is a 2-to-1 and $x_i = x_i \oplus 111$ for all $i \in \{0, 1\}^3$, so $s = 101$

(a) Give the state after measuring the second register (suppose $|001\rangle$)

(b) Give the state after final Hadamards

(c) Suppose the first run of the algorithm gives $j = 001$, and the second gives $j = 101$. Show that assuming $s \neq 000$, those two runs of the algorithm already determine $s$. 
Problem 5 Analysis Technique Proof: 5 pts

Show that $e^{iAx} = \cos(x)I + i\sin(x)A$ where $A^2 = I$ and $x$ is some real number.
Problem 6  QFT : 5+5 pts

(1) Please show that

\[ QFT_N |x⟩ = \frac{1}{\sqrt{N}} (|0⟩ + e^{\frac{2\pi i x}{N}} |1⟩) \otimes \cdots \otimes (|0⟩ + e^{\frac{2\pi i x}{N}} |1⟩) \]

where \(|x⟩ = |x_1 \cdots x_n⟩\) and \(N = 2^n\)

(2) Please draw the circuit where \(n = 3\).
Problem 7  Superdense Coding: 5 + 5 pts

(1) Draw the circuit for superdense coding

(2) Verify how Alice sends the message 10 to Bob using the above circuit.